STAT414, Semester I 2009-2010

Assignment 11

(Deadline: 11/20/2009)

1. (Ex8.7) Suppose that X follows a geometric distribution,

$$P(X = k) = p(1-p)^{k-1}$$

and assume an iid sample of size n.

   (a) Find the method of moments estimate of p.

   (b) Find the MLE of p.

2. (Ex8.16) Consider an iid sample of random variables with density function

$$f(x|\sigma) = \frac{1}{2\sigma}\exp(-\frac{|x|}{\sigma}),$$

   (a) Find the method of moments estimate of σ.

   (b) Find the MLE of σ.

3. (EX8.19) Suppose that $X_1, X_2, \ldots, X_n$ are iid $N(\mu, \sigma^2)$.

   (a) If μ is known, what is the MLE of σ?

   (b) If σ is known, what is the MLE of μ?

   (c) In the case above (σ known), does any other unbiased estimate of μ have smaller variance?

4. Suppose that $X_1, \ldots, X_n$ form a random sample from a distribution for which the pdf $f(x|\theta)$ is as follows:

$$f(x|\theta) = \begin{cases} \theta x^{\theta-1} & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Suppose also that the value of the parameter θ is unknown ($\theta > 0$), and the prior distribution of θ is a gamma distribution with parameters α and β ($\alpha > 0$ and $\beta > 0$). Determine the mean and the variance of the posterior distribution of θ.

5. Suppose that the proportion θ of defective items in a large shipment is unknown, and the prior distribution of θ is a beta distribution for which the parameters are $\alpha = 5$ and $\beta = 10$.

Suppose also that 20 items are selected at random from the shipment, and that exactly one of these items is found to be defective. If the squared error loss function is used, what is the Bayes estimate of θ?

6. Suppose that a random sample is to be taken from a normal distribution for which the value of the mean θ is unknown and the standard deviation is 2, the prior distribution of θ is a normal distribution for which the standard deviation is 1, and the value of θ must be estimated by using the squared error loss function. What is the smallest random sample that must be taken in order for the mean squared error of the Bayes estimator of θ to be 0.01 or less?