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STAT 604
Intro. to Statistical Computing

Law of Large Number, Central Limit Theorem, and Monte Carlo Simulation

Let X_1, \dots, X_n be independent and identically distributed random variables with expectation $\mu < \infty$ and variance $\sigma^2 < \infty$. Let:

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i.$$

The (Strong) **Law of Large Numbers** (LLN) states that:

$$\bar{X}_n \xrightarrow{a.s.} \mu, \text{ as } n \rightarrow \infty.$$

The **Central Limit Theorem** states (CLT) that:

$$\sqrt{n} (\bar{X}_n - \mu) \rightarrow_d N(0, \sigma^2), \text{ as } n \rightarrow \infty.$$

Monte Carlo Application: If we want to estimate the mean of the distribution, we can simulate draws from the distribution (using the computer), use the sample mean to estimate the mean of the distribution, and assess our uncertainty about the true mean using a confidence interval based on the normal distribution.

Note: This strategy works for any other quantities of interest about the distribution (i.e., not just the mean).