Stata Software for Measurement Error Models

This material was put together by Roberto G. Gutierrez of the Stata Corporation. Users should consult http://www.stata.com/merror/ as the web site for this software.

1 Installing the software

Installing the software of Hardin & Carroll (2003) is very easy, and requires Stata version 8.0 or later (the current version as of the publication of our book is Stata 9.1). From a “net-aware” Stata, simply type

```
  . net install http://www.stata-journal/software/sj3-4/st0049
```
which installs the following Stata programs

- **qvf** – Generalized linear models with (possibly) instrumental variables and fast bootstrap
- **rcal** – Generalized linear models with regression calibration
- **simex** – Simulation extrapolation in generalized linear models
- **simexploit** – SIMEX plots after estimation with **simex**

If you find the exact website hard to remember, you can use Stata’s net searching tools to find the software. The Stata command, `findit`, is very useful in this regard, e.g.

```
  . findit simex
```
or

```
  . findit measurement error models
```
will allow you to then point-and-click your way towards installing the appropriate software.

2 Getting the Framingham Data

The Framingham data are in Stata format, saved as `framingham.dta`. It can be loaded into Stata, described, and partially listed as follows

```
  . use framingham, clear
  (Framingham Heart Study)
  . describe
  Contains data from framingham.dta
  obs: 1,615 Framingham Heart Study
  vars: 14 20 Jan 2006 09:31
  size: 62,985 (99.4% of memory free) (_dta has notes)

  variable name storage display value variable label
    type format label
  age   byte  %9.0g age at exam 2
  sbp21 int  %9.0g first systolic blood pressure at exam 2
  sbp22 int  %9.0g second systolic blood pressure at exam 2
  sbp31 int  %9.0g first systolic blood pressure at exam 3
  sbp32 int  %9.0g second systolic blood pressure at exam 3
```

Sorted by:

notes

_dta:

list firstchd age smoke cholest3 in 1/10

<table>
<thead>
<tr>
<th>firstchd</th>
<th>age</th>
<th>smoke</th>
<th>cholest3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>56</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>38</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>54</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>42</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>47</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>43</td>
<td>1</td>
</tr>
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<td>7</td>
<td>0</td>
<td>58</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
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<td>43</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>43</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>36</td>
<td>0</td>
</tr>
</tbody>
</table>

The dataset is titled *Framingham Heart Study*. It should be available on the Stata website for direct download within Stata. That is, readers should be able to type

use http://www.stata-press.com/data/nlmem/framingham, clear

and they would then have the data ready for analysis in Stata. The directory name *nlmem* stands for Non-linear measurement error models.
3 Analysis With One Error-Prone Covariate, Known Error Variance

3.1 Background

- Variables $Z$ measured without error: age, smoke (smoking status) and cholest3 (Cholesterol)
- Variable $W$ measured with known measurement error: lsbp3 (Transformed systolic blood pressure)
- Measurement error variance: 0.1260

3.2 Naive Analysis

We begin with a naive analysis, treating lsbp3 as if it were measured exactly. In Stata, this analysis can be done with the commands logistic or glm, but here we use qvf which, when used without instrumental variables, is analogous to glm with option irls for iterated re-weighted least squares.

```
.qvf firstchd age smoke cholest3 lsbp3, family(binomial)
```

![Generalized linear models](image)

- Coef. Std. Err. $z$ P>|z| [95% Conf. Interval]
- age 0.05683 0.0117455 4.84 0.000 0.0338092 0.0798508
- smoke 0.5731081 0.2497074 2.30 0.022 0.0836907 1.062526
- cholest3 0.0078488 0.0021162 3.71 0.000 0.003701 0.0119965
- lsbp3 1.524242 0.3887089 3.92 0.000 0.7623869 2.286098
- _cons -14.1818 1.786893 -7.94 0.000 -17.68404 -10.67955

noting that we simply need to specify family(binomial) since the logit link is canonical (and the default) for that family.

The standard errors above are analytical. If we instead wanted bootstrap standard errors, we add options bootstrap and seed(10001), the latter necessary only for reproducibility.

```
.w qvf firstchd age smoke cholest3 lsbp3, family(binomial) bootstrap seed(10001)
```

![Bootstrap](image)

- Coef. Std. Err. $z$ P>|z| [95% Conf. Interval]
- age 0.05683 0.0111139 5.11 0.000 0.0350473 0.0786128
- smoke 0.5731081 0.2337721 2.45 0.014 0.1149231 1.031293
- cholest3 0.0078488 0.0019552 4.01 0.000 0.0040167 0.0116808
- lsbp3 1.524242 0.2337772 6.04 0.000 1.0684887 1.979996
- _cons -14.1818 1.51011 -9.39 0.000 -17.14154 -11.22205
For all the software covered here, the default number of bootstrap replications is 199, but this can be overruled via option `brep()`, e.g. `brep(500)`.

### 3.3 Regression Calibration

For a regression calibration analysis, we use command `rcal`. In this example, we assume the measurement error variance is known and thus we need to define the known variance as a Stata matrix and pass this matrix to `rcal` by using option `suuinit()`.

```
. matrix U = 0.01260 // define the m.e. variance
. rcal (firstchd = age smoke cholest3) (wlsbp:lsbp3), family(binomial) suuinit(> U)
```

#### Regression calibration

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Link</td>
<td>Logit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residual df</td>
<td>1610</td>
<td>Wald F(4,1610)</td>
<td>17.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Prob &gt; F</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Scale param</td>
<td>.908332</td>
</tr>
<tr>
<td>Variance Function:</td>
<td>V(u) = u(1-u) [Bernoulli]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Link Function :</td>
<td>g(u) = log(u/(1-u)) [Logit]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### firstchd Coef. Std. Err. t P>|t| [95% Conf. Interval]

- **age**
  - Coef.: .0537147
  - Std. Err.: .0114134
  - t: 4.71
  - P>|t|: 0.000
  - [95% Conf. Interval]: (0.031328, 0.0761014)

- **smoke**
  - Coef.: .5816822
  - Std. Err.: .2380915
  - t: 2.44
  - P>|t|: 0.015
  - [95% Conf. Interval]: (0.1146804, 1.048884)

- **cholest3**
  - Coef.: .0075968
  - Std. Err.: .0020234
  - t: 3.75
  - P>|t|: 0.000
  - [95% Conf. Interval]: (0.0036281, 0.0115655)

- **wlsbp**
  - Coef.: 2.047573
  - Std. Err.: .5001969
  - t: 4.09
  - P>|t|: 0.000
  - [95% Conf. Interval]: (1.066468, 3.028678)

- **_cons**
  - Coef.: -16.26699
  - Std. Err.: 1.703024
  - t: -9.55
  - P>|t|: 0.000
  - [95% Conf. Interval]: (-19.60736, -12.92661)

In the above, note the slightly different syntax for specifying the model: the response and covariates measured without error are in the first equation, separated by parentheses, while the single variable measured with error, `lsbp3`, comprises the second equation, labelled as `wlsbp`.

As was the case last time, the above are analytical standard errors. Switching to bootstrap standard errors works just as before.

```
. rcal (firstchd = age smoke cholest3) (wlsbp:lsbp3), family(binomial) suuinit(> U)
    bootstrap seed(10002)
```

#### Regression calibration

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Link</td>
<td>Logit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residual df</td>
<td>1610</td>
<td>Wald F(4,1610)</td>
<td>20.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Prob &gt; F</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Scale param</td>
<td>.908332</td>
</tr>
<tr>
<td>Variance Function:</td>
<td>V(u) = u(1-u) [Bernoulli]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Link Function :</td>
<td>g(u) = log(u/(1-u)) [Logit]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Bootstrap firstchd Coef. Std. Err. t P>|t| [95% Conf. Interval]

- **age**
  - Coef.: .0537147
  - Std. Err.: .0108181
  - t: 4.97
  - P>|t|: 0.000
  - [95% Conf. Interval]: (0.0324956, 0.0749338)

- **smoke**
  - Coef.: .5816822
  - Std. Err.: .2412495
  - t: 2.41
  - P>|t|: 0.016
  - [95% Conf. Interval]: (0.1084861, 1.054878)

- **cholest3**
  - Coef.: .0075968
  - Std. Err.: .0020114
  - t: 3.78
  - P>|t|: 0.000
  - [95% Conf. Interval]: (0.0036516, 0.0115421)

- **wlsbp**
  - Coef.: 2.047573
  - Std. Err.: .5125924
  - t: 3.99
  - P>|t|: 0.000
  - [95% Conf. Interval]: (1.042154, 3.052992)

- **_cons**
  - Coef.: -16.26699
  - Std. Err.: 2.189779
  - t: -7.43
  - P>|t|: 0.000
  - [95% Conf. Interval]: (-20.5621, -11.97187)

### 3.4 SIMEX

Analysis using `simex` is achieved using the `simex` command. The syntax is identical to that for `rcal`, with the exception that analytical standard errors are not available. By default `simex` will
not calculate any standard errors, but we can request bootstrap standard errors as previously

```
. simex (firstchd = age smoke cholest3) (wlsbp:lsetp3), family(binomial) suuinit
> (U) bootstrap seed(10003)
Estimated time to perform bootstrap: 2.28 minutes.
```

Simulation extrapolation

<table>
<thead>
<tr>
<th>No. of obs</th>
<th>Wald F(4,1610)</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1615</td>
<td>20.66</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Residual df = 1610

Variance Function: \( V(u) = u(1-u) \) [Bernoulli]

Link Function : \( g(u) = \log(u/(1-u)) \) [Logit]

| firstchd | Coef.  | Std. Err. | t     | P>|t|  | [95% Conf. Interval] |
|----------|--------|-----------|-------|------|----------------------|
| age      | 0.0541254 | 0.0120696 | 4.48  | 0.000 | 0.0304516 - 0.0777992 |
| smoke    | 0.5822821 | 0.2290522 | 2.54  | 0.011 | 0.1330103 - 1.031554 |
| cholest3 | 0.007754  | 0.002152  | 3.60  | 0.000 | 0.003533 - 0.0119751 |
| wlsbp    | 1.854374  | 0.4400707 | 4.21  | 0.000 | 0.991202 - 2.717545  |
| _cons    | -15.48957 | 1.834077  | -8.45 | 0.000 | -19.087 - 11.89214  |

Once `simex` has been run, we can examine the extrapolant function with command `simexplot`

```
. simexplot wlsbp
```

producing the graph in Figure 1.

![Simex plot for proposed analysis #1](image_url)
The quadratic extrapolant function is the default for simex, with linear and rational being options. In this example, but the non-linear (rational linear) curve fit refused to converge and estimation reverted to the quadratic extrapolant.
4 Analysis With One Error-Prone Covariates, Replicates

4.1 Background

- Variables \(Z\) measured without error: \texttt{age}, \texttt{smoke}, \texttt{cholest3}
- Variable \(W\) measured with unknown measurement error: \texttt{wlsbp} with two replicate measurements \texttt{lsbp2} and \texttt{lsbp3}
- Measurement error variance: unknown, estimated from the replicates

4.2 Naive Analysis

For the naive analysis, we want the average of the replicates, \texttt{lsbp2} and \texttt{lsbp3}, to be the variable measured without error, in which case we need to generate this new variable first

```
. gen mlsbp = (lsbp2 + lsbp3) / 2
. qvf firstchd age smoke cholest3 mlsbp, family(binomial)
```

| firstchd | Coef. | Std. Err. | z     | P>|z|  | [95% Conf. Interval] |
|----------|--------|-----------|-------|------|---------------------|
| age      | 0.0554057 | 0.0118037 | 4.69  | 0.000 | 0.0322709, 0.0785405 |
| smoke    | 0.5930756 | 0.2499898 | 2.37  | 0.018 | 0.1031045, 1.083047  |
| cholest3 | 0.0078725 | 0.0021084 | 3.73  | 0.000 | 0.0037402, 0.0120048 |
| mlsbp    | 1.706105  | 0.4174073 | 4.09  | 0.000 | 0.8880022, 2.524209  |
| _cons    | -14.94906 | 1.899273  | -7.87 | 0.000 | -18.67156, -11.22655 |

for analytical standard errors. Switching to bootstrap
4.3 Regression Calibration

For regression calibration, note that the equation labelled as \( wlsbp \) now has two variables in it, representing the replicate measurements. Since we estimate the measurement error by using the replicates, the option `suuinit()` is no longer necessary.

\[
\text{. rcal (firstchd = age smoke cholest3) (wlsbp: lsbp2 lsbp3), family(binomial)}
\]

Regression calibration  
No. of obs = 1615
Link = Logit
Residual df = 1610  
Wald F(4,1610) = 17.85
Prob > F = 0.0000
Scale param = .913985
Variance Function: \( V(u) = u(1-u) \)  
[Logit]
Link Function : \( g(u) = \log(u/(1-u)) \)

| firstchd | Coef. | Std. Err. | t | P>|t| | [95% Conf. Interval] |
|----------|-------|-----------|---|------|----------------------|
| age      | .0535814 | .0114046  | 4.70 | 0.000 | .031212 – .0759508  |
| smoke    | .6012368 | .2391313  | 2.51 | 0.012 | .3219253 – 1.070278 |
| cholest3 | .0077435 | .0020176  | 3.84 | 0.000 | .0038217 – .0117009 |
| wlsbp    | 2.010981 | .4718496  | 4.26 | 0.000 | 1.085477 – 2.936485 |
| _cons    | -16.1729 | 1.815754  | -8.91 | 0.000 | -19.73439 – -12.61141 |

At this point we mention that, whenever appropriate, you can add option `robust` to get Sandwich estimates of standard errors. This is fairly standard throughout all of Stata.

(Continued on next page)
. rcal (firstchd = age smoke cholest3) (wlsbp:lsbp2 lsbp3), family(binomial) ro bust
Regression calibration
Link = Logit
Residual df = 1610
Wald F(4,1610) = 21.65
Prob > F = 0.0000
(IRLS EIM) Scale param = .913985
Variance Function: V(u) = u(1-u) [Bernoulli]
Link Function : g(u) = log(u/(1-u)) [Logit]

| firstchd  | Semi-Robust Coef. | Std. Err. | t   | P>|t| | [95% Conf. Interval] |
|-----------|-------------------|-----------|-----|------|---------------------|
| age       | .0535814          | .0107486  | 4.98| 0.000| .0324986 .0746642  |
| smoke     | .6012368          | .2442884  | 2.46| 0.014| .12208 1.080394   |
| cholest3  | .0077435          | .0019718  | 3.93| 0.000| .0038759 .0116112 |
| wlsbp     | 2.010981          | .4636976  | 4.34| 0.000| 1.101467 2.920496 |
| _cons     | -16.1729          | 1.735387  | -9.32| 0.000| -19.57676 -12.76905|

And, finally, bootstrap standard errors

. rcal (firstchd = age smoke cholest3) (wlsbp:lsbp2 lsbp3), family(binomial) bs trap seed(10005)
Regression calibration
No. of obs = 1615
Bootstrap reps = 199
Residual df = 1610
Wald F(4,1610) = 23.15
Prob > F = 0.0000
(IRLS EIM) Scale param = .913985
Variance Function: V(u) = u(1-u) [Bernoulli]
Link Function : g(u) = log(u/(1-u)) [Logit]

| firstchd  | Bootstrap Coef. | Std. Err. | t   | P>|t| | [95% Conf. Interval] |
|-----------|-----------------|-----------|-----|------|---------------------|
| age       | .0535814        | .0106838  | 5.02| 0.000| .0326257 .0745371  |
| smoke     | .6012368        | .2218764  | 2.71| 0.007| .1860399 1.036434  |
| cholest3  | .0077435        | .001876  | 4.13| 0.000| .0040639 .0114231 |
| wlsbp     | 2.010981        | .4641468  | 4.33| 0.000| 1.100586 2.921377  |
| _cons     | -16.1729        | 1.982635  | -8.16| 0.000| -20.06172 -12.28409|

4.4 SIMEX
For simex we have

. simex (firstchd = age smoke cholest3) (wlsbp:lsbp2 lsbp3), family(binomial) b strap seed(10006)
Estimated time to perform bootstrap: 2.25 minutes.
Simulation extrapolation
No. of obs = 1615
Bootstrap reps = 199
Residual df = 1610
Wald F(4,1610) = 21.61
Prob > F = 0.0000
Variance Function: V(u) = u(1-u) [Bernoulli]
Link Function : g(u) = log(u/(1-u)) [Logit]

| firstchd  | Bootstrap Coef. | Std. Err. | t   | P>|t| | [95% Conf. Interval] |
|-----------|-----------------|-----------|-----|------|---------------------|
| age       | .0537021        | .0112988  | 4.75| 0.000| .0315403 .075864   |
| smoke     | .5991448        | .2520173  | 2.38| 0.018| .1048283 1.093461  |
| cholest3  | .0077896        | .0020684  | 3.77| 0.000| .0037324 .0118467  |
| wlsbp     | 1.905424        | .4326691  | 4.40| 0.000| 1.05677 2.754078   |

9
and plotted with

```
    . simexplot wlsbp
```

producing the graph in Figure 2.

![Simex plot for proposed analysis #2](image.png)

Figure 2: Simex plot for proposed analysis #2
5 Analysis With Two Error-Prone Covariates, Known Error Covariance Matrix

5.1 Background and Important Calling Convention

In this analysis, we have two variables measured with error, namely transformed systolic blood pressure and log-cholesterol. In the example, the measurement error covariance matrix is assumed known.

There is an **important convention** here: we input the measurement error covariance matrix so that the diagonals correspond to the measurement error variances for transformed systolic blood pressure first, and then log-cholesterol. When calling regression calibration and SIMEX, the variables must be entered exactly in the same order, or strange results will occur.

- Variables $Z$ measured without error: age, smoke
- Variables $W$ measured with known measurement error: lsbp3 and cholest3
- Measurement error variance known:

$$V = \begin{bmatrix} 0.0126020814393 & 0.00067287539939 \\ 0.00067287539939 & 0.00845894714763 \end{bmatrix}$$

5.2 Naive Analysis

```
. qvf firstchd age smoke lcholest3 lsbp3, family(binomial)
```

|                | Coef. | Std. Err. | z     | P>|z|  | [95% Conf. Interval] |
|----------------|-------|-----------|-------|------|---------------------|
| age            | 0.056446 | 0.0117413 | 4.81  | 0.000 | 0.0334334 .0794585  |
| smoke          | 0.572659 | 0.2498046 | 2.29  | 0.022 | 0.0830509 1.062267 |
| lcholest3      | 2.039176 | 0.5435454 | 3.75  | 0.000 | 0.9738468 3.104506 |
| lsbp3          | 1.518676 | 0.3889605 | 3.90  | 0.000 | 0.7583275 2.281025 |
| _cons          | -23.39799 | 3.413942 | -6.85 | 0.000 | -30.0892 -16.70679 |

With bootstrap standard errors:

```
. qvf firstchd age smoke lcholest3 lsbp3, family(binomial) bootstrap seed(10001)
```

|                | Coef. | Std. Err. | z     | P>|z|  | [95% Conf. Interval] |
|----------------|-------|-----------|-------|------|---------------------|
| age            | 0.056446 | 0.0111898 | 5.04  | 0.000 | 0.0345143 .0783776  |
5.3 Regression calibration

```
. mat V = (0.01260208144393, 0.00067287539939 \ 0.00067287539939, 0.00845894714763)
. rcal (firstchd = age smoke) (wlsbp:lsbp3) (wcholest:lcholest3), family(binomial) suuinit(V)
```

Regression calibration
No. of obs = 1615

```
  Link = Logit
  Residual df = 1610
  Wald F(4, 1610) = 17.63
  Prob > F = 0.0000
  (IRLS EIM)
  Scale param = 0.906104
  Variance Function: V(u) = u(1-u) [Bernoulli]
  Link Function: g(u) = log(u/(1-u)) [Logit]
```

```
  firstchd Coef. Std. Err. t P>|t| [95% Conf. Interval]
  age .0526627 .0113888 4.62 0.000 .0303243 .0750012
  smoke .5774215 .2379919 2.43 0.015 .1106151 1.044228
  wlsbp 2.014178 .4997267 4.03 0.000 1.033995 2.994361
  wcholest 2.768454 .7114252 3.89 0.000 1.373038 4.163871
  _cons -29.33517 3.249715 -9.03 0.000 -35.70928 -22.96105
```

With bootstrap standard errors:

```
. rcal (firstchd = age smoke) (wlsbp:lsbp3) (wcholest:lcholest3), family(binomial) suuinit(V) bootstrap seed(10007)
```

Regression calibration
No. of obs = 1615

```
  Bootstrap reps = 199
  Residual df = 1610
  Wald F(4, 1610) = 21.74
  Prob > F = 0.0000
  (IRLS EIM)
  Scale param = 0.906104
  Variance Function: V(u) = u(1-u) [Bernoulli]
  Link Function: g(u) = log(u/(1-u)) [Logit]
```

```
  firstchd Coef. Std. Err. t P>|t| [95% Conf. Interval]
  age .0526627 .01113 4.73 0.000 .0308319 .0744936
  smoke .5774215 .2429321 2.38 0.018 .1009251 1.050293
  wlsbp 2.014178 .4997267 4.01 0.000 1.033995 2.994361
  wcholest 2.768454 .7114252 3.89 0.000 1.373038 4.163871
  _cons -29.33517 3.858431 -7.60 0.000 -36.90324 -21.76709
```

5.4 SIMEX

```
. simex (firstchd = age smoke) (wlsbp:lsbp3) (wcholest:lcholest3), family(binomial) suuinit(V) bootstrap seed(10008)
```

Simulation extrapolation
No. of obs = 1615

```
  Bootstrap reps = 199
  Residual df = 1610
  Wald F(4, 1610) = 24.10
  Prob > F = 0.0000
```

Estimated time to perform bootstrap: 2.40 minutes.
Variance Function: $V(u) = u(1-u)$  [Bernoulli]
Link Function : $g(u) = \log(u/(1-u))$  [Logit]

| firstchd | Coef.     | Std. Err. | t    | P>|t| | [95% Conf. Interval] |
|----------|-----------|-----------|------|------|----------------------|
| age      | 0.0545443 | 0.0099631 | 5.47 | 0.000 | 0.0350023 0.0740863  |
| smoke    | 0.5803764 | 0.2638591 | 2.20 | 0.028 | 0.0628329 1.09792   |
| wlsbp    | 1.84699   | 0.4529421 | 4.08 | 0.000 | 0.9585718 2.735408  |
| wcholest | 2.5346    | 0.7278619 | 3.48 | 0.001 | 1.106944 3.962256   |
| _cons    | -27.44831 | 4.231603  | -6.49| 0.000 | -35.74834 -19.14828|

SIMEX plots:

. simexplot wcholest

Figure 3: Simex plot for proposed analysis #3, wcholest

. simexplot wlsbp

and the graphs produced are included at the end of this document. Finally, note that the two final graphs may be combined into one.
Figure 4: Simex plot for proposed analysis #3, *wlsbp*