STAT 612 **MIDTERM EXAM** **NOVEMBER 4, 1999**

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1. Consider the three-factor model where each factor has two levels and there is only one observation per cell. The model has no three-way interaction and only one two-way interaction (Factors A & B).

(a) Write out $\mathbf{X}$ and $\underline{\beta}$ for the overparameterized model **AND** give a set of constraints that will allow for a solution to the normal equations.

(b) Write out the cell means model.

(c) Find the $\mathbf{H}$ matrices for the null hypothesis of no main effect for Factor A for both the overparameterized model and the cell means model.

(d) Find an unbiased estimator of σ^2. By find, I mean specify the specific $\mathbf{A}$ matrix and scalar m such that $\underline{Y}'\mathbf{A}\underline{Y}/m$ is your estimator. (To get the maximum points, you should give the estimator with the largest possible degrees of freedom.)

2. Consider a two-factor cell means model with each factor having only two levels and only one observation per cell. The model has the following constraints: $\mu_{11} = \mu_{22}$ and $\mu_{12} = \mu_{21}$. Let $\underline{Y}' = (8, 6, 4, 7)$.

(a) Display $\underline{Y}_R$, $\mathbf{W}_R$ and $\underline{\mu}_R$.

(b) Estimate $\underline{\mu}$ under the model with the additional constraint that $\mu_{11} - \mu_{12} = 1$.

3. Consider the linear model $\underline{Y} = \mathbf{X}\underline{\beta} + \underline{e}$ where $\underline{e} \sim \text{N}(0,\ \sigma^2\mathbf{I})$ and and $\mathbf{X}$ is $n \times p$ of rank p.

(a) Partition $\mathbf{X}$ into $[\mathbf{X_1}|\mathbf{X_2}]$ and show that SSE_F, the error sum of squares for the model $\underline{Y} = \mathbf{X}\underline{\beta}+\underline{e}$, can not be larger than SSE_R, the error sum of squares for the model $\underline{Y} = \mathbf{X_1}\underline{\beta}_1 + \underline{e}$.

(b) Under the full model, find the distribution of SSE_R, as defined in part (a).

4. Let $\underline{Y} \sim \text{N}(\underline{\mu},\ \Sigma)$, where $\underline{\mu}' = (3,\ 3,\ 3)$ and $\Sigma = \mathbf{2I} + \mathbf{J}$. Let $U_i = \underline{Y}'\mathbf{A_i}\underline{Y},\ i = 1,\ 2,\ 3;$ where

$$\mathbf{A_1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{A_2} = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}.$$

(a) Are U_1 and U_2 independent?

(b) Does there exist a matrix $\mathbf{A_3}$ such that U_3 has a $\chi^2_{(2,\lambda)}$ distribution *and* is independent of U_2? If so, find one. If not, why not?