STAT 612 **HOMEWORK NO. 7** **DUE: December 5, 2005**

1. Consider the model $Y_{ijk} = \mu + \alpha_i + b_j + \alpha b_{ij} + c_k + e_{ijk}$, where $i = 1, 2$, $j = 1, 2, 3, 4$ and $k = 1, 2, 3$, and all random effects have the usual Normal distributions.

(a) Is $\alpha_1 - \alpha_2$ estimable? If yes, why? If no, are there any conditions under which it would be estimable?

(b) Find the REML estimates of the variance components when $SS_\alpha = 2$, $SS_b = 6$, $SS_{\alpha b} = 9$, $SS_c = 6$ and $SS_e = 28$.

(c) Show that the mean square error of the MOM estimator of σ^2_C, $\hat{\sigma}^2_C$, is greater than the mean square error of the *adjusted* MOM estimator $\tilde{\sigma}^2_C = max(0, \hat{\sigma}^2_C)$.
RECALL: $\text{MSE}(\hat{\sigma}^2_c) = \int_{-\infty}^{\infty} (\hat{\sigma}^2_c - \sigma^2_c)^2 f_{\hat{\sigma}^2_c}(\hat{\sigma}^2_c) d\hat{\sigma}^2_c$.

2. Consider the saturated four-factor mixed model where Factors A & B are fixed and Factors C & D are random. Assume that each factor has 3 levels and that there are 2 observations per cell.

(a) Determine the quadratic form for the ABC interaction. Finds its distribution.

(b) Determine a test to test the $\text{H}_o : \sigma^2_{cd} = 0$.

(c) Determine an estimator of the variance of the estimate of $\overline{Y}_{1....} - \overline{Y}_{2....}$. What is $\overline{Y}_{1....} - \overline{Y}_{2....}$ estimating (in an unbiased fashion)?

(d) Write out the Source, DF and EMS columns for the resulting ANOVA table.

3. Consider the saturated three-factor model with Factor A being fixed and Factors B & C being random. Assume all factors has 2 levels and that there are 3 observations per cell.

(a) Determine a test statistics to test H_o : *No Effect of Factor A*.

(b) Determine a confidence interval for $\alpha_1 - \alpha_2$.