

1. Let \mathbf{S} be a positive semidefinite matrix. Assume that \mathbf{S} is $p \times p$ of rank $p - 1$, where the last row of \mathbf{S} is a linear combination of the preceding rows.
 - (a) Determine the form of \mathbf{T} , the Cholesky decomposition of \mathbf{S} .
 - (b) Explain how the Cholesky decomposition algorithm can be used to find a g-inverse for \mathbf{S} and, assuming that $\mathbf{S} = \mathbf{X}'\mathbf{X}$, how it can be used to find an estimate of $\underline{\ell}'\underline{\beta}$.

2. Consider the three factor model with Factors A and B being crossed and Factor C nested within Factors A & B. Assume that Factor A has 2 levels, Factor B 3 levels, Factor C 4 levels and that there are two observations per cell.
 - (a) Write out the overparameterized model.
 - (b) Determine \mathbf{X} and $\underline{\beta}$ for this model.
 - (c) Determine the quadratic forms and degrees of freedom for each term in the ANOVA table.
 - (d) Show that the sums-of-squares add the SST.
 - (e) Determine the expected value of SSC and its distribution.

3. Consider the two-factor nested model, where Factor A has 2 levels, Factor B has 3 levels and there are 2 observations per cell.
 - (a) Determine \mathbf{X} and $\underline{\beta}$.
 - (b) Find the normal equations.
 - (c) Determine a set of non-estimable functions that can be used to solve the normal equations.
 - (d) Assume that $\underline{Y}' = (3, 4, 5, 4, 2, 2, 5, 9, 7, 6, 8, 5)$. Find a solution to the normal equations.
 - (e) Use the reduced \mathbf{X} matrix and PROC REG to fit the appropriate null and alternative models to construct the ANOVA table for these data.

4. Consider the three-factor crossed model where the only interactions are the AB and AC interactions. All factors have 2 levels and there are 2 observations per cell.
 - (a) Write out the overparameterized model.
 - (b) Write out the cell means model (using matrix notation).
 - (c) Determine the quadratic form needed to compute SSE.
 - (d) Determine the reduced cell means model.
 - (e) Determine the null hypothesis for testing for no effect for the AB interaction under the reduced model.