STAT 612 **HOMEWORK NO. 4** **DUE: October 10, 2005**

1. Consider the model $\underline{Y} = \mathbf{X}\underline{\beta} + \underline{e}$, where $\underline{e} \sim \mathrm{N}(\underline{0}, \sigma^2\mathbf{I})$ and

$$\mathbf{X} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 3 \\ 1 & 4 & 2 & 1 \\ 1 & 2 & 0 & 0 \\ 1 & 1 & 2 & 3 \\ 1 & 3 & 2 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \text{ and } \underline{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}.$$

(a) Find the reduced model under the constraint that $\beta_1 + \beta_2 = 1 + \beta_3$. Write out $\mathbf{G}$, $\underline{g}$, $\underline{Y}_R$, $\mathbf{X}_R$ and $\underline{\beta}_R$.

(b) Find the reduced model under the constraints that $\beta_1 = \beta_2 = \beta_3$. Write out $\mathbf{G}$, $\underline{g}$, $\underline{Y}_R$, $\mathbf{X}_R$ and $\underline{\beta}_R$.

(c) Based on the reduced model in part (b), find $\widehat{\underline{\beta}}$ when $\underline{Y}' = (5, 3, 8, 9, 2, 3, 1)$.

(d) Based on part (c), determine an unbiased estimate of σ^2.

2. Consider the 3 quadratic forms $U_i = \underline{Y}' A_i \underline{Y}$; i=1,2,3, where the U_i are mutually independent, $U_i/\delta_i \sim \chi^2_{(r_i,0)}$ and

$$\mathrm{E}(U_1/r_1) = \delta_1 = \sigma_1^2 + \sigma_3^2,\ \ \mathrm{E}(U_2/r_2) = \delta_2 = \sigma_2^2 + \sigma_3^2 \text{ and } \mathrm{E}(U_3/r_3) = \delta_3 = \sigma_3^2.$$

(a) Find unbiased estimators ($\widehat{Q}_1$ & $\widehat{Q}_2$) of $Q_1 = \sigma_1^2 + \sigma_2^2 + 2\sigma_3^2$ and $Q_2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$.

(b) Use the Satterthwaite approach to find an approximate distributions for $\widehat{Q}_1$ and $\widehat{Q}_2$.

(c) Simulate 1,000 replicates of $\widehat{Q}_1$, find the empirical distribution and plot the Satterthwaite approximation over the empirical distribution to see how well they match. (Let $r_1 = r_2 = r_3 = 5$ and use $\sigma_1^2 = .6$, $\sigma_2^2 = .2$ and $\sigma_3^2 = .15$.)

(d) Simulate 1,000 replicates of $\widehat{Q}_2$, find the empirical distribution and plot the Satterthwaite approximation over the empirical distribution to see how well they match. (Let $r_1 = r_2 = r_3 = 2$ and use $\sigma_1^2 = .02$, $\sigma_2^2 = .02$ and $\sigma_3^2 = .50$.)

3. Consider the linear regression model, $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + e_i$.

(a) Would SST(Reduced)=SST(Full) when testing H: $\beta_1 + 4\beta_2 = 0$?

(b) Would SST(Reduced)=SST(Full) when testing H: $\beta_0 = 5$?

(c) Would SST(Reduced)=SST(Full) when testing H: $\beta_1 + 4\beta_2 - \beta_0 = 0$?

4. Show that $(\mathbf{X}'\mathbf{X})^{-}\mathbf{X}'\underline{Y}$ is a solution to the normal equations when $\mathbf{X}$ is not full column rank.

5. State and prove the Guass-Markov Theorem under the model $\underline{Y} = \mathbf{X}\underline{\beta} + \underline{e}$ with $\mathbf{G}\underline{\beta} = \underline{g}$.