STAT 612 **FINAL EXAM** **DECEMBER 15, 1999**

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1. Consider the model $Y_{ijk} = \mu + \alpha_i + b_j + c_{ik} + e_{ijk}$, where $i = 1, 2,\ j = 1, 2$ and $k = 1, 2$. Assume that the b_j, c_{ik} and e_{ijk} are all independent and normally distributed with variances of σ_b^2, σ_c^2 and σ_e^2, respectively.

(a) Display Σ_Y.

(b) Determine the SS matrices for SS_b, SS_c and SS_e.

(c) Find the EMS for the quadratic forms determined in part (b).

(d) Assuming that $SS_b = 10$, $SS_c = 16$ and $SS_e = 27$, find both the OLS and REML estimates of the associated variance components.

2. Let $\underline{Y} = \mathbf{X}\underline{\beta} + \underline{e}$, where $\underline{e} \sim (0,\, \sigma^2\mathbf{I})$. Assume that $\underline{\beta} = (\beta_0,\, \beta_1)'$ and that $\mathbf{X} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \end{pmatrix}$.

(a) Find the variance of $\widehat{\beta}_1$, the OLS estimator of β_1.

(b) Prove or disprove that $\tilde{\beta}_1 = (Y_3 - Y_1)/3$ is unbiased, but has a variance that is larger than the variance of $\widehat{\beta}_1$.

3. Let $Y_{ijk} = \mu_{ij} + e_{ijk}$, where the e_{ijk} are iid N(0,σ^2), $i = 1, 2$ and $j = 1, 2, 3$. Let $n_{ij} = n$, for all (i, j), except (1,1) and (2,3), where $n_{ij} = 0$. The model has two constraints. $\mu_{11} - \mu_{21} = \mu_{12} - \mu_{22}$ and $\mu_{12} - \mu_{22} = \mu_{13} - \mu_{23}$

(a) Write out the reduced model.

(b) Determine the effective hypothesis for $\text{H}_o : \mu_{1\cdot} = \mu_{2\cdot}$.

(c) Assume the two constraints in this problem were replaced by following three constraints: $\mu_{11} = \mu_{12}$, $\mu_{11} + \mu_{12} - 2\mu_{13} = 0$ and $\mu_{11} - \mu_{12} - \mu_{21} + \mu_{22} = 0$. Is the design still connected? Explain.

4. Consider an experiment designed to study 2 different brands of fertilizer at 3 different doses (0 lb, 1 lb, 2 lbs). Each brand/dose combination (cell) will contain 1 observation. Note, however, that when the dose is 0 lb, no fertilizer is actually being applied. In that cell, none of the two brands is being observed. So, there only 5 cells, each with only one observation.

(a) Write out the cell means model and *then* define the concept of no interaction for this experiment.

(b) Find the reduced cell means model under the assumption of no interaction.

(c) Let $\underline{Y} = (2, 4, 7, 6, 8)'$. Starting with the reduced model from part b, estimate $\underline{\mu}$ under the additional constraints that $\mu_{11} + \mu_{12} = \mu_{21} + \mu_{22}$ and $\mu_{11} + \mu_{21} = \mu_{12} + \mu_{22}$.