STAT 612 **FINAL EXAM** **DECEMBER 14, 1998**

SHOW YOUR WORK AND START EACH PROBLEM ON A NEW PAGE

1. Consider the three factor completely crossed design with no three-factor interaction. Factors A and B have two levels and Factor C has three levels. There are two observations per cell except in cells n_{111} and n_{112}, which are empty.

(a) Write out the cell means model with any necessary contraints **AND** determine the effective model, after adjusting for the empty cells.

(b) Specify the **H** matrix for $\mathrm{H}_o : \textit{No } A \times B \textit{ interaction}$ under the effective model.

(c) Find the reduced model under the hypothesis defined in part (b). Specify the degrees of freedom for the error term under the reduced model. (You can leave the matrices in formula form *if* you explicitly define each of the matrices in the formula.)

2. Let $\underline{Y} \sim \mathrm{N}(\mathbf{X}\underline{\beta}, \Sigma)$ where $\mathbf{X} = X^* \otimes \underline{1}_2$ with X^* being a 10×2 matrix that has a column of ones in its first column. Let $\Sigma = \sigma^2[(\mathbf{I}_{10} \otimes \mathbf{I}_2) + (\mathbf{I}_{10} \otimes \mathbf{J}_2)] = \sigma^2[\mathbf{I}_{10} \otimes (\mathbf{I}_2 + \mathbf{J}_2)]$. Note $\Sigma^{-1} = \sigma^{-2}[\mathbf{I}_{10} \otimes (\mathbf{I}_2 - \frac{1}{3}\mathbf{J}_2)]$.

(a) Derive the m.l.e. of $\underline{\beta}$ and σ^2. (Do not simply provide a general formula.)

(b) Is $\widehat{\underline{\beta}}$ independent of $\sum_{i=1}^{10} \sum_{j=1}^{2} (Y_{ij} - \overline{Y}_{i\cdot})^2$, where $\overline{Y}_{i\cdot} = 2^{-1} \sum_{j=1}^{2} Y_{ij}$?

3. Consider a mixed linear model where Factors A and B are crossed and Factor C is nested within Factor B. Let Factors A and B be fixed and Factor C be random. There is an $A \times B$ interaction, but there is no $A \times C$ interaction. All three factors have two levels, but there is only one observation per cell.

(a) Write out the overparameterized model and determine $\mathrm{E}(\underline{Y})$ and Σ_Y.

(b) Determine A_c and A_e for SSC and SSE and derive the Expected Mean Squares for C and error.

(c) Derive $\widehat{\sigma}_c^2$, the unbiased M.O.M. estimators for σ_c^2, and prove it has a mean square error that is greater than or equal to that of the mean square error of $\tilde{\sigma}_c^2 = max(0, \widehat{\sigma}_c^2)$.

4. Consider two models for Y where for both models e_i is iid $\mathrm{N}(0, \sigma^2)$ and $[\mathbf{X_1}|\mathbf{X_2}]$ in $n \times (p_1 + p_2)$ of rank $p_1 + p_2$.

M1: $\underline{Y} = \mathbf{X}_1\underline{\beta}_1 + \mathbf{X}_2\underline{\beta}_2 + \underline{e}$.

M2: $\underline{Y} = \mathbf{X}_1\underline{\beta}_1 + \underline{e}$.

(a) Under what conditions does the estimator of $\underline{\beta}_1$ using **M1** equal the estimator of $\underline{\beta}_1$ using **M2**?

(b) Prove that the SSE under **M1** is less than or equal to the SSE under **M2**.