Confidence Intervals and Decision Making

Estimation of an unknown parameter, say μ, can be accomplished either using a point estimator or an interval estimator. A point estimator (a formula used to produce an estimate) yields a single value. For example, in the case of μ, the point estimator is $\bar{X}$. The drawback with point estimators is that computing the estimate does not give you any insight into the accuracy or quality of the answer. Some measure of precision is needed. How this is done depends upon what is being estimated. In the case of μ, we can use s^2/n to help. The most common way to combine the point estimator with the estimator of precision is to produce a confidence interval. The confidence interval produces a range within which we feel the parameter is most likely to be located. The smaller the interval is the greater the precision. The formula for a $(1-\alpha)*100\%$ confidence interval for μ is:

$\bar{X} \pm t_{\alpha/2,\, n-1}(s^2/n)^{1/2}$, where $t_{\alpha/2,\, n-1}$ is found from the t-table in Table A.4.

Interpretation of a, say 95%, confidence interval is often done incorrectly. A confidence interval is a range of values within which the data suggest the true population parameter is likely to be located. It is not a guarantee that the true parameter is in the interval, however. That is where the 95% comes in. As you will recall, in most cases, there is an unbelievable number of possible samples of size *n* one could take from a population. The 95% refers to the fact that if you repeated the process of constructing a confidence interval for all those possible samples, 95% of the resulting interval would include the right answer. The other 5% would miss the correct location for the parameter of interest.

A natural application of confidence intervals, other than to simply state where you think μ is located, is to answer questions about which values of μ are still believable given the data that have been observed. Any value within the interval, regardless of where it is in the interval, is still a plausible value that is consistent with the data observed. Values outside the interval are not.

The size of the confidence interval is impacted by the variance and the sample size. We can not choose the variance, but we can choose the sample size. Given a plausible value for the variance, σ^2, we can determine the size sample needed to produce a confidence interval of a desired length. If we want the $(1-\alpha)*100\%$ confidence interval to have a specific width of 2E, that is $\bar{X} \pm E$, then the required sample size is:

$$n=([z_{\alpha/2} * \sigma]/E)^2.$$