\[ (1 - \rho^2) = 1 - \frac{2\sigma_x \sigma_y \rho}{\sigma_x^2 + \sigma_y^2 + 2ab} \]

where
\[
\sigma_x = \sqrt{1 - \rho^2}.
\]

We can then write
\[
\begin{align*}
    f_{U,V}(u,v) &= \frac{1}{2\pi \sigma_x \sigma_y \sqrt{1 - \rho^2}} \exp \left( -\frac{1}{2(1-\rho^2)} \left( \frac{(u - \mu_x)^2}{\sigma_x^2} + \frac{(v - \mu_y)^2}{\sigma_y^2} \right) \right) \\
    &= \frac{1}{2\pi \sqrt{1 - \rho^2}} \exp \left( -\frac{1}{2(1-\rho^2)} \left( \frac{(u - \mu_x)^2}{\sigma_x^2} + \frac{(v - \mu_y)^2}{\sigma_y^2} \right) \right)
\end{align*}
\]

which is in the exact form of a bivariate normal distribution. Thus, by part a), \( U \) is normal.

4.42 a)
\[
\begin{align*}
    E X &= a_X E Z_1 + b_X E Z_2 + E c_X = a_X \mu_0 + b_X \mu_0 + c_X = c_X. \\
    Var X &= a_X^2 Var Z_1 + b_X^2 Var Z_2 + Var c_X = a_X^2 + b_X^2. \\
    E Y &= a_Y \mu_0 + b_Y \mu_0 + c_Y = c_Y. \\
    Var Y &= a_Y^2 Var Z_1 + b_Y^2 Var Z_2 + Var c_Y = a_Y^2 + b_Y^2. \\
    Cov(X,Y) &= E XY - EX \cdot EY \\
    &= E(a_X a_Y Z_1^2 + b_X b_Y Z_2^2 + c_X c_Y Z_1 Z_2 + a_X b_Y Z_1 Y_2 + b_X c_Y Z_1 Y_2) \\
    &= a_X a_Y \sigma_x \sigma_y + b_X b_Y \sigma_x \sigma_y + c_X c_Y \mu_0 - c_X c_Y \mu_0 \\
    &= a_X a_Y \sigma_x \sigma_y + b_X b_Y \sigma_x \sigma_y.
\end{align*}
\]

since \( E Z_1^2 = E Z_2^2 = 1 \), and expectations of other terms are all zero.

b) Simply plug the expressions for \( a_X, b_X, \) etc. into the equalities in a) and simplify.

4.43 a) By definition of \( Z \), for \( z < 0 \),
\[
P(Z \leq z) = P(X \leq z \text{ and } XY > 0) + P(-X \leq z \text{ and } XY < 0)
\]

which by symmetry and independence, is negative.

b) By definition of \( Z \),
\[
\begin{align*}
    P(Z > 0) &= P(X > 0 \text{ and } Y > 0) + P(X < 0 \text{ and } Y < 0) \\
    &= P(X \leq 0) P(Y > 0) + P(X > 0) P(Y \leq 0) \\
    &= P(X \leq 0) P(Y > 0) \\
    &= P(X \leq 0) P(Y < 0) \\
    &= P(X \leq z) P(Y < 0) \\
    &= P(X \leq z) P(Y > 0) \\
    &= P(X \leq z).
\end{align*}
\]

By a similar argument, for \( z > 0 \), we get \( P(Z > z) = P(X > z) \), and hence, \( P(Z \leq z) = P(X \leq z) \). Thus, \( Z \sim N(1,1) \).

4.49 i) \( X > 0 \text{ and } Y > 0 \)

ii) \( X < 0 \text{ and } Y > 0 \)

So \( Z \) and \( Y \) have the same sign, hence they cannot be bivariate normal.

4.50 Let \( X_1, X_2, X_3 \) be independent exponential(\( \lambda \)) random variables, and let \( Y = \max(X_1, X_2, X_3) \), the lifetime of the system. Then
\[
P(Y \leq y) = P(\max(X_1, X_2, X_3) \leq y) = P(X_1 \leq y) P(X_2 \leq y) P(X_3 \leq y),
\]

by the independence of \( X_1, X_2, \) and \( X_3 \). Now each probability is
\[
P(X_1 \leq y) = \int_0^y \frac{1}{\lambda} e^{-x/\lambda} dx = 1 - e^{-y/\lambda},
\]

\[
P(X_2 \leq y) = \int_0^y \frac{1}{\lambda} e^{-x/\lambda} dx = 1 - e^{-y/\lambda},
\]

and
\[
P(X_3 \leq y) = \int_0^y \frac{1}{\lambda} e^{-x/\lambda} dx = 1 - e^{-y/\lambda}.
\]